LINEAR PROGRAMMING

MATH 8430

Course Description:

This course includes a complete development of theoretical and computational aspects of linear programming. Basic theoretical foundations covered include polyhedra, convexity, linear inequalities and duality. Advanced topics such as decomposition and column generation are covered. Both simplex methods and interior point methods are included. **3 credits**

Prerequisites:

Math 4300/8306

Overview of Content and Purpose of the Course:

Students will gain a thorough knowledge of the theoretical aspects of linear programming and understand the mathematical foundations of the simplex method. They will learn techniques such as decomposition and column generation for dealing with large-scale problems. They will understand computational aspects of linear programming as it relates to both simplex and interior point methods.

Anticipated Audience/Demand:

Graduate students in mathematics, computer science, and engineering.

Major Topics:

1) Introduction
- **a.** Linear Programming Model
- **b.** Applications
- **c.** Geometric Solution

2) Simultaneous Linear Equations

3) Convexity

4) Polyhedra
- **a.** Polyhedral Sets and Cones
- **b.** Extreme Points, Extreme Directions
- **c.** Representation of Polyhedral Sets

5) Simplex Method
- **a.** Extreme Points and Optimality
- **b.** Basic Feasible Solutions
- **c.** Algebra of the Simplex Method
- **d.** Optimality and Unboundedness
- **e.** Simplex Method (Tableaus)
- **f.** Revised Simplex Method
- **g.** Block Pivoting

6) Starting Solution and Convergence

- **a.** Initial Basic Feasible Solution
- **b.** Two-Phase Method
- **c.** Big-M Method
- **d.** Degeneracy
- **e.** Cycling

7) Duality and Sensitivity

- **a.** Formulation of the Dual Problem
- **b.** Primal-Dual Relationships
- **c.** Economic Intepretation of the Dual
- **d.** Dual Simplex
- **e.** Primal-Dual Algorithm
- **f.** Sensitivity Analysis

8) Advanced simplex topics

- **a.** Product Form of the Inverse
- **b.** Decomposition
- **c.** Column Generation

9) Computational Complexity of Linear Programming

- **a.** Worst-Case Behavior of the Simplex Method
- **b.** Ellipsoid Method
- **c.** Primal-Dual Interior Point Method

Methods:

This course will be presented by lecture and class discussions.

Student Role:

Students must attend and participate in class and must complete the course requirements.

Textbook:

M. Bazarraa, J. Jarvis, and H. Sherali. *Linear Programming and Network Flows*, Wiley , 2004.
S. Nash and A. Sofer. *Linear and Nonlinear Programming*, McGraw Hill, 1996.

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